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Model-independent search for the Abelian Z' gauge boson virtual state based on the new observable in the processes $e^+e^- \to l^+l^-$ is analyzed for the LEP data. The observable is introduced by using following from renormalizability of an underlying theory the correlations between the parameters of the low energy effective Lagrangian and kinematics of the scattering processes. The central values of the observable are in accordance with the Z' existence, although no real signal is found at the 1σ level. Lower limits at the 95% confidence level on the corresponding contact interaction scale are derived in the range of 15.6 - 18.6 TeV. Comparisons with other fits are done.

I. INTRODUCTION

Among various objectives of the recently finished LEP experiments an important place was devoted to searching for signals of new physics beyond the energy scale of the standard model (SM). Reports on these results are adduced partly in the literature [1]. In the present note we are going to discuss the problem of searching for the heavy Z' gauge boson [2]. This particle is a necessary element of the different models extending the SM. Low limits on its mass following from the analysis of variety of popular models (χ, ψ, η, L–R models [3] and the Sequential Standard Model (SSM) [4]) are found to be in the energy intervals 500–2000 GeV [1] (see Table I which reproduces Table 9 of Ref. [1]). As it is seen, the values of $m_{Z'}$ (as well as the parameters of interactions with the SM particles) are strongly model dependent. Therefore, it seems reasonable to find some model independent signals of this particle. To elaborate that general principles of field theory must be taken into consideration giving a possibility to relate the parameters of different scattering processes. Then, one is able to introduce variables, convenient for the model independent search for Z' (or other heavy states). These ideas were used in Ref. [5] in order to introduce the model-independent sign definite variables for Z' detection in scattering processes with $\sqrt{s} \simeq 500$ GeV.

As it has been pointed out in Ref. [5], some parameters of new heavy fields can be related by using the requirement of renormalizability of the underlying model remaining in other respects unspecified. The relations between the parameters of new physics due to the renormalizability were called the renormalization group (RG) relations. In Ref. [5] the RG relations for the low-energy Z' couplings to the SM fields have been derived. They predict two possible types of Z' particles, namely, the chiral and the Abelian Z' ones. Each type is described by a few couplings to the SM fields. Therefore, taking into account the RG correlations between the Z' couplings, one is able to introduce observables which uniquely pick up the Z' virtual state [5]. In the present paper we discuss these observables and the constraints on possible Z' signals following from the analysis of the LEP data.

II. Z' COUPLINGS TO FERMIONS

To consider the Z' interactions with light particles one must specify the model describing physics at low energies. For instance, the minimal SM with the one scalar doublet can be chosen. However, due to the lack of information about scalar fields, models with an extended set of light scalar particles can be also considered. Below we choose the two-Higgs-doublet model (THDM) [6] as the low-energy theory (notice, the minimal SM is the particular case of the THDM).

To derive the RG relations one has to introduce the parametrization of Z' couplings to the SM fields. Since we are going to account of the Z' effects in the low-energy $e^+e^- \to \bar{f}f$ processes in lower order in $m_{Z'}^{-2}$, the linear in Z' interactions with the SM fields are of interest, only. The renormalizability of the underlying theory and the decoupling theorem [7] guarantee the dominance of renormalizable Z' interactions at low energies. The interactions of the non-renormalizable types, generated at high energies due to radiation corrections, are suppressed by the inverse heavy mass. The SM gauge group $SU(2)_L \times U(1)_Y$ is considered as a subgroup of the underlying theory group. So, the mixing interactions of the types $Z'W^+W^-$, $Z'ZZ$, ... are absent at the tree level. Such conditions are usually used in the literature [8] and lead to the following parametrization of the linear in Z' low-energy vertices:

$$\mathcal{L} = \sum_{i=1}^{2} \left| \left(D_\mu^{\mathrm{ew},\phi} - \frac{i\tilde{g}}{2}\tilde{Y}(\phi_i)\tilde{B}_\mu \right) \phi_i \right|^2 + i \sum_{f=f_L,f_R} \bar{f}\gamma^\mu \left(D_\mu^{\mathrm{ew},f} - \frac{i\tilde{g}}{2}\tilde{Y}(f)\tilde{B}_\mu \right) f, \qquad (1)$$

where ϕ_i $(i=1,2)$ are the scalar doublets, $\tilde{g}$ is the charge corresponding to the Z' gauge group, $D_\mu^{\mathrm{ew},\phi}$ and $D_\mu^{\mathrm{ew},f}$

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are the electroweak covariant derivatives, $\tilde{B}_\mu$ denotes the massive Z' field before the spontaneous breaking of the electroweak symmetry, and the summation over the all SM left-handed fermion doublets, $f_L = \{(f_u)_L, (f_d)_L\}$, and the right-handed singlets, $f_R = (f_u)_R, (f_d)_R$, is understood. Diagonal 2×2 matrices $\tilde{Y}(\phi_i)$, $\tilde{Y}(f_L)$ and numbers $\tilde{Y}(f_R)$ are unknown Z' generators characterizing the model beyond the SM.

The one-loop RG relations for the above introduced Z' vertices (1) have been obtained in Ref. [5]. As it was shown, two different types of the Z' generators are compatible with the renormalizability of the underlying theory. The first type, called the chiral Z', describes the Z' boson which couples to the SM doublets, only. The corresponding generators have the zero traces:

$$\begin{aligned} \tilde{Y}(\phi_i) &= -\tilde{Y}_\phi \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\ \tilde{Y}(f_L) &= \tilde{Y}_{L,f_u} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\ \tilde{Y}(f_R) &= 0. \end{aligned} \tag{2}$$

The second type is the Abelian Z' boson:

$$\begin{aligned} \tilde{Y}(\phi_i) &= \tilde{Y}_\phi \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\ \tilde{Y}(f_L) &= \tilde{Y}_{L,f} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\ \tilde{Y}(f_R) &= \tilde{Y}_{L,f} + 2T^3_f \tilde{Y}_\phi, \end{aligned} \tag{3}$$

where T^3_f is the third component of the fermion weak isospin. The relations (3) ensure, in particular, the invariance of the Yukawa terms with respect to the effective low-energy $\tilde{U}(1)$ subgroup corresponding to the Z' boson. As it follows from the relations, the couplings of the Abelian Z' to the axial-vector fermion currents have the universal absolute value proportional to the Z' coupling to the scalar doublets.

The derived relations (2)–(3) are independent of a specific model beyond the SM predicting the Z' boson. They hold in the THDM as well as in the minimal SM. As it is seen from relations (2)–(3), only one parameter for each SM doublet remains arbitrary. The rest parameters are expressed through them. A few number of independent Z' couplings gives the possibility to introduce the observables convenient for detecting uniquely the Z' signals in experiments [5,9]. In what follows, we analyze the obtained at LEP data taking into account the RG relations (3) in order to constrain possible signals of the Abelian Z' boson.

III. OBSERVABLES FOR THE ABELIAN Z' SEARCH

Consider the leptonic processes $e^+e^- \to V^* \to l^+l^-$ $(l = \mu, \tau)$ with the neutral vector boson exchange ($V = A, Z, Z'$). We assume the non-polarized initial- and final-state fermions. At LEP energies $\sqrt{s} \simeq 200$ GeV the fermions can be treated as massless particles, $m_e, m_l \sim 0$. In this approximation the left-handed and the right-handed fermions can be substituted by the helicity states, which we mark as λ and ξ for the incoming electron and the outgoing fermion, respectively ($\lambda, \xi = L, R$).

The differential cross section of the process $e^+e^- \to V^* \to l^+l^-$ deviates from its SM value by a quantity of order $m_{Z'}^{-2}$:

$$\begin{aligned} \Delta \frac{d\sigma_l}{d\cos\theta} &= \frac{1}{16\pi s} \mathrm{Re}\Big[\mathcal{A}^*_{\mathrm{SM}} \\ &\quad \times \left(\mathcal{A}_{Z'} + \left.\frac{d\mathcal{A}_Z}{d\theta_0}\right|_{\theta_0=0} \theta_0\right)\Big], \\ \mathcal{A}_{\mathrm{SM}} &= \mathcal{A}_A + \mathcal{A}_Z(\theta_0 = 0), \end{aligned} \tag{4}$$

where θ denotes the angle between the momentum of the incoming electron and the momentum of the outgoing lepton, $\mathcal{A}_V$ is the Born amplitude of the process, and $\theta_0 \sim m_W^2/m_{Z'}^2$ is the Z–Z' mixing angle. The leading contribution comes from the interference between the Z' exchange amplitude, $\mathcal{A}_{Z'}$, and the SM amplitude, $\mathcal{A}_{\mathrm{SM}}$, whereas the Z–Z' mixing terms are suppressed by the additional small factor m_Z^2/s. Notice that the deviation $\Delta d\sigma_l/d\cos\theta$ depends on the center-of-mass energy through the quantity m_Z^2/s, only.

To take into consideration the correlations (3) let us introduce the observable $\sigma_l(z)$ defined as the difference of cross sections integrated in some ranges of the scattering angle θ, which will be specified below [5,9]:

$$\begin{aligned} \sigma_l(z) &\equiv \int_z^1 \frac{d\sigma_l}{d\cos\theta} d\cos\theta - \int_{-1}^z \frac{d\sigma_l}{d\cos\theta} d\cos\theta \\ &= \sigma_l^T \left[A_l^{FB}\left(1 - z^2\right) - \frac{z}{4}\left(3 + z^2\right)\right], \end{aligned} \tag{5}$$

where z stands for the boundary angle, σ_l^T denotes the total cross section and A_l^{FB} is the forward-backward asymmetry of the process. The idea of introducing the z-dependent observable (5) is to choose the value of the kinematic parameter z in such a way that to pick up the characteristic features of the Abelian Z' signals. Due to the correlations between the Abelian Z' couplings the quantity (5) can be written as follows

$$\begin{aligned} \Delta\sigma_l(z) &= \frac{\alpha_{\mathrm{em}}\tilde{g}^2}{16 m_{Z'}^2} \left(\mathcal{F}_0^l(z,s) a_{Z'}^2\right. \\ &\quad + \mathcal{F}_1^l(z,s) v_{Z'}^l v_{Z'}^e + \mathcal{F}_2^l(z,s) v_{Z'}^l |a_{Z'}| \\ &\quad \left. + \mathcal{F}_3^l(z,s) v_{Z'}^e |a_{Z'}|\right). \end{aligned} \tag{6}$$

where $v_{Z'}^l \equiv (\tilde{Y}_{L,l} + \tilde{Y}_{R,l})/2$ and $a_{Z'}^l \equiv (\tilde{Y}_{R,l} - \tilde{Y}_{L,l})/2 = T_f^3 \tilde{Y}_\phi$ are the Z' couplings to the vector and the axial-vector lepton currents. Functions $\mathcal{F}_i^l(z,s)$ are determined by the SM quantities and independent of the lepton generation. The leading contributions to the factors

$\mathcal{F}_2^l(z,s) = \mathcal{F}_3^l(z,s)$ equal to zero. So, by choosing the boundary angle z^* to be the solution to the equation $\mathcal{F}_1^l(z^*,s) = 0$, one can switch off three factors $\mathcal{F}_i^l(z,s)$ $(i = 1, 2, 3)$ simultaneously. The function $z^*(s)$ is the decreasing function of energy. This is shown in Table II for the LEP energies. In Table II we also show the factors $\mathcal{F}_i^l(z^*,s)$. As it is occurred, these factors contribute less than 2%.

By choosing $z = z^*$, we obtain the sign definite observable

$$\Delta\sigma_l(z^*) \simeq \frac{\alpha_{\mathrm{em}}\tilde{g}^2}{16 m_{Z'}^2}\mathcal{F}_0^l(z^*,s) a_{Z'}^2 < 0. \qquad (7)$$

The quantity $\Delta\sigma_l(z^*)$ is negative and the same for the all types of the SM charged leptons. This is the model-independent signal of the Abelian Z' boson. Thus, the variables $\Delta\sigma_l(z^*)$ select the Z' boson signals in the processes $e^+e^- \to l^+l^-$.

IV. ANALYSIS OF THE LEP DATA

The measurements of the cross-sections and the forward-backward asymmetries have been combined for the full LEP2 data set recently. The preliminary results are adduced in Ref. [1]. Let us analyze these data assuming a model independent search for signals of the Abelian Z' boson.

Before doing that in the way outlined in the previous sections, we note that in Ref. [1] some other analysis of signals of physics beyond the SM in the processes $e^+e^- \to \bar{f}f$ was present. It is based on the introduction of the "models" describing different kinds of new four-fermion contact interactions. Since one is able to successfully fit only one parameter of new physics, eight models (LL, RR, LR, RL, VV, AA, A0, V0) assuming various helicity coupling between the initial state and final state currents are discussed. Each model is described by only one non-zero coupling. For example, in the LL model the non-zero coupling of left-handed fermions is taken into account. The signal of a new heavy particle is fitted by considering the interference of the SM amplitude with the contact four-fermion term. Whatever physics beyond the SM exists, it can manifest itself in some contact coupling mentioned. Hence, it is possible to find a low limit on the masses of the states responsible for the interactions considered. In principle, a number of states may contribute into each of the models. Therefore, the purpose of the fit described by these models is to find any signal of new physics. No specific types of new particles are considered in this analysis. The virtual states of heavy particles (for instance, the Abelian Z' boson) contribute to several contact interactions simultaneously, and the corresponding couplings cannot be switched off separately.

It is interesting to note the fits for the process $e^+e^- \to \mu^+\mu^-$ in Ref. [1]. Several models mentioned demonstrate one standard deviation from the SM predictions for this process. In this regard, we note the paper [10] in which the mentioned models were investigated for the Bhabha scattering $e^+e^- \to e^+e^-(\gamma)$. The deviations from the SM at the 1σ–level were found again, whereas the AA model shows the 2σ–level deviation. However, the question whether these deviations could be interpret as a signal of the Abelian Z' remains open.

The aim of our investigation is to pick up a model-independent signal of the virtual Abelian Z' boson, which uniquely selects this particle. The strategy is similar - to introduce the proper one-parametric variable (7) on the discussed principles of renormalizability and the kinematic properties of scattering amplitudes. In our case we have a possibility to derive this observable by taking into consideration the renormalizability relations (3) and, therefore, the selection of the axial-vector currents is not the "model". The observable $\Delta\sigma_l(z^*)$ accounts for all interactions of the Z' boson. Below, we will make an additional comparison of our method and that of in Ref. [1].

Now, to search for the model-independent Z' signals we will analyze the observable $\Delta\sigma_l(z^*)$ on the base of the full LEP data set. It depends on one unknown Z' parameter, $\tilde{g}^2 a_{Z'}^2/m_{Z'}^2$, which describes the effective contact interaction between the axial-vector lepton currents. In what follows we will use the notation

$$\frac{\tilde{g}^2 a_{Z'}^2}{16\pi m_{Z'}^2} \equiv \frac{1}{\Lambda^2} \equiv \epsilon. \qquad (8)$$

This normalization is admitted, in particular, in Ref. [1]. The parameter ϵ is related to the observable $\Delta\sigma_l(z^*)$ by the factor which slightly depends on energy (see Table II):

$$\epsilon = \frac{\Delta\sigma_l(z^*)}{\pi\alpha_{\mathrm{em}}\mathcal{F}_0^l(z^*,s)}. \qquad (9)$$

Treating the observable $\Delta\sigma_l(z^*)$ has the following advantages:

1. All the LEP data for the processes $e^+e^- \to l^+l^-$ can be incorporated to obtain the limits on the observable.

2. There is one parameter of new physics to be fitted.

3. The data for the $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ scattering can be used to measure the same observable.

4. The sign of the observable ($\epsilon > 0$) is the characteristic feature of the Abelian Z' signal.

The LEP data for the total cross-sections and the forward-backward asymmetries as well as the SM values of these quantities are shown in Tables III–IV. From the set of the data we compute the observable $\Delta\sigma_l(z^*)$ and the corresponding error $\delta\sigma_l(z^*)$ for each LEP energy by means of the following relations

$$
\begin{aligned}
\Delta\sigma_l(z^*) &= \left[A_l^{FB}\left(1-z^{*2}\right) - \frac{z^*}{4}\left(3+z^{*2}\right)\right]\Delta\sigma_l^T \\
&\quad + \left(1-z^{*2}\right)\sigma_{l,\mathrm{SM}}^T \Delta A_l^{FB}, \\
\delta\sigma_l(z^*)^2 &= \left[A_l^{FB}\left(1-z^{*2}\right) - \frac{z^*}{4}\left(3+z^{*2}\right)\right]^2 (\delta\sigma_l^T)^2 \\
&\quad + \left[\left(1-z^{*2}\right)\sigma_{l,\mathrm{SM}}^T\right]^2 (\delta A_l^{FB})^2.
\end{aligned} \tag{10}
$$

The results are given in Table V and Fig. 1. All the values of the observable are no more than one standard deviation from the SM value except for two points at 161 and 172 GeV corresponding to the $e^+e^- \to \tau^+\tau^-$ process. These points reflect the significant dispersion of the measurements for the scattering into τ pairs at $\sqrt{s} <$ 183 GeV. As it is also seen from Fig. 1, the measurements for the center-of-mass energies $\sqrt{s} \geq 183$ GeV have a higher level of precision.

We will choose three different sets of data to fit the parameter ϵ. The first one is the complete set containing 20 points. The second set includes the data for the center-of-mass energies $\sqrt{s} \geq 183$ GeV (12 measurements). The final set contains the data for the scattering into μ pairs (10 points).

Thus, there are 20 measurements ϵ_i ($i = 1, \ldots, 20$) of the parameter ϵ with unequal precisions. We define the fitted value of ϵ, $\bar{\epsilon}$, as a linear combination of ϵ_i with factors which minimize the corresponding dispersion, $(\delta\bar{\epsilon})^2$. It is easy to show that

$$
\bar{\epsilon} = \left[\sum_i \frac{1}{(\delta\epsilon_i)^2}\right]^{-1} \sum_i \frac{\epsilon_i}{(\delta\epsilon_i)^2}, \tag{11}
$$

$$
(\delta\bar{\epsilon})^2 = \left[\sum_i \frac{1}{(\delta\epsilon_i)^2}\right]^{-1}. \tag{12}
$$

The value of $\delta\bar{\epsilon}$ gives the 1σ-level interval for $\bar{\epsilon}$. Note that the same value of $\bar{\epsilon}$ can be derived as a result of the minimization of the likelihood function

$$
-\log\mathcal{L}(\epsilon) = \sum_i \frac{(\epsilon_i - \epsilon)^2}{2(\delta\epsilon_i)^2}, \tag{13}
$$

or the corresponding χ^2-function. In this case the 1σ-level interval $(\bar{\epsilon}-\delta\bar{\epsilon}, \bar{\epsilon}+\delta\bar{\epsilon})$ could be alternatively derived from the condition $\log[\mathcal{L}(\epsilon)/\mathcal{L}(\bar{\epsilon})] = -0.5$.

We use the log-likelihood method to determine a one sided lower limit on the scale Λ at the 95% confidence level. It is derived by the integration of the likelihood function over the physically allowed region $\epsilon > 0$. The exact definition is

$$
\int_0^{1/\Lambda^2} \mathcal{L}(\epsilon')d\epsilon' = 0.95 \int_0^\infty \mathcal{L}(\epsilon')d\epsilon'. \tag{14}
$$

In Table VI we show the fitted values of ϵ with their 68% confidence level uncertainties and the 95% confidence level lower limit on the scale Λ. We also compute the total probability of the Z' signal by integrating the likelihood function over the whole allowed range of ϵ ($\epsilon > 0$).

As it is seen, all data sets lead to the comparable fitted values of $\bar{\epsilon}$ with the nearly equal uncertainties. All the central values $\bar{\epsilon}$ have the sign compatible with the Z' signal. However, they are no more than one standard deviation from the SM value $\bar{\epsilon} = 0$. Thus, though the central values witness to the Z' existence, no Z' signal is detected at the 1σ confidence level. This result reflects the lack of the accuracy of the input data. In this regard, fitting the observable directly from the final differential cross sections when they will be published may probably improve the accuracy.

The more precise data corresponding to the scattering into $\mu^+\mu^-$ pairs demonstrate the largest positive central value of ϵ. This value is nearly one standard deviation. Taking into account the data for $\tau^+\tau^-$ final states decreases the central value of the Z' signal but does not affect essentially the uncertainty of the results.

As our investigation showed, the characteristic signal of the Abelian Z' boson is concerned with the coupling of axial-vector currents. In this regard, let us turn again to the helicity "models" of Ref. [1] and compare our results with the fitting for the AA case. As it follows from the present analysis this model is sensitive mostly to the signals of the Abelian Z' boson. Of course, the parameter ϵ in Ref. [1] and in Eq. (5) is not the same quantity. As we already noted, in the AA model the Z' couplings to the vector fermion currents are set to zero, therefore it is able to describe only some particular case of the Abelian Z' boson. Moreover, in this model both the positive and the negative values of ϵ are considered, whereas in our approach only the positive ones are permissible. As the value of four-fermion contact coupling in the AA model is dependent on the lepton flavor, the Abelian Z' induces the axial-vector coupling universal for all lepton types. Nevertheless, it is interesting to note that the fitted value of ϵ in the AA model for the $\mu^+\mu^-$ final states ($-0.0033^{+0.0032}_{-0.0012}$ TeV^{-2}) as well as the value derived under the assumption of the lepton universality ($-0.0013^{+0.0018}_{-0.0015}$ TeV^{-2}) are similar to our results. Since the parameters ϵ in Ref. [1] and in Eq. (8) have opposite signs by definition, the signs of the central values in the AA model agree with those of Table VI, whereas the uncertainties are of the same order. Thus, as it follows from this analysis, the AA model is mainly responsible for signals of the Abelian Z' gauge boson although a lot of details concerning its interactions is not accounted for within this fit.

The Z' boson mass is related to the contact interaction scale Λ by Eq. (8). To convert the scale Λ to the Z' mass one have to assume the value of coupling $\tilde{g}$. When the Z' boson couples to the SM particles with a strength comparable with the electroweak forces $\tilde{g} \simeq g$, the central values of $\bar{\epsilon}$ correspond to the masses of order 3–5 TeV, whereas the lower limit on $m_{Z'}$ is about 1.4–1.7 TeV. Now let us compare these values with the constraints for different models in Table I. As it is seen, the Abelian Z' cannot be so light as the χ, ψ, and η models predict. On the other hand, the lower limits derived in our paper are close to ones of the L–R models and the Sequential Standard Model. Thus, although the Z' boson is not detected at LEP, it can be light enough to be discovered at LHC.

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TABLE I. 95% confidence level lower limits on the Z' mass for some popular models.

Model	χ	ψ	η	L–R	SSM
$m_{Z'}^{\rm limit}$, GeV/c^2	630	510	400	950	2260

TABLE II. The boundary angle z^* and the factors $\mathcal{F}_i^l(z^*,s)$ computed at energies of the LEP experiments.

$\sqrt{s}$, GeV	z^*	$\mathcal{F}_0^l(z^*,s)$	$\mathcal{F}_{2,3}^l(z^*,s)$
130	0.488	-2.349	-0.051
136	0.466	-2.456	-0.053
161	0.409	-2.381	-0.049
172	0.393	-2.300	-0.047
183	0.381	-2.225	-0.045
189	0.376	-2.187	-0.045
192	0.374	-2.169	-0.044
196	0.370	-2.146	-0.044
200	0.368	-2.124	-0.043
202	0.366	-2.114	-0.043

TABLE III. The LEP data for σ_l^T (pb) and A_l^{FB}

$\sqrt{s}$, GeV	σ_l^T	$\delta\sigma_l^T$	A_l^{FB}	δA_l^{FB}
		$l=\mu$		
130	8.331	0.664	0.736	0.059
136	8.229	0.678	0.709	0.062
161	4.585	0.364	0.546	0.07
172	3.555	0.317	0.679	0.077
183	3.484	0.147	0.558	0.035
189	3.108	0.077	0.565	0.021
192	2.925	0.177	0.538	0.052
196	2.948	0.107	0.585	0.032
200	3.052	0.107	0.522	0.032
202	2.622	0.14	0.541	0.048
		$l=\tau$		
130	9.065	0.927	0.652	0.084
136	7.123	0.821	0.771	0.088
161	5.692	0.545	0.769	0.063
172	4.026	0.45	0.344	0.107
183	3.398	0.174	0.609	0.045
189	3.14	0.1	0.584	0.028
192	2.86	0.219	0.608	0.068
196	2.994	0.141	0.498	0.046
200	2.966	0.137	0.553	0.043
202	2.8	0.186	0.583	0.059

TABLE IV. SM values for σ_l^T (pb) and A_l^{FB}

$\sqrt{s}$, GeV	$\sigma_\mu^{T,\rm SM}$	$\sigma_\tau^{T,\rm SM}$	$A_\mu^{FB,\rm SM}$	$A_\tau^{FB,\rm SM}$
130	8.439	8.435	0.705	0.704
136	7.281	7.279	0.684	0.683
161	4.613	4.613	0.609	0.609
172	3.952	3.951	0.591	0.591
183	3.446	3.446	0.576	0.576
189	3.207	3.207	0.569	0.569
192	3.097	3.097	0.566	0.566
196	2.962	2.962	0.562	0.562
200	2.834	2.833	0.558	0.558
202	2.77	2.769	0.556	0.556

TABLE V. $\Delta\sigma_l(z^*)$ (pb) and ϵ (TeV^{-2}) from the LEP data

$\sqrt{s}$, GeV	$\Delta\sigma_l(z^*)$	$\delta\sigma_l(z^*)$	ϵ	$\delta\epsilon$
		$l=\mu$		
130	0.181	0.395	-0.0081	0.0176
136	0.313	0.374	-0.0133	0.0159
161	-0.246	0.273	0.0108	0.0120
172	0.189	0.270	-0.0086	0.0123
183	-0.046	0.106	0.0022	0.0050
189	-0.030	0.060	0.0014	0.0029
192	-0.104	0.142	0.0050	0.0068
196	0.056	0.085	-0.0027	0.0041
200	-0.053	0.080	0.0026	0.0040
202	-0.063	0.118	0.0031	0.0058
		$l=\tau$		
130	-0.270	0.548	0.0120	0.0244
136	0.466	0.536	-0.0198	0.0228
161	0.957	0.298	-0.0421	0.0131
172	-0.826	0.358	0.0376	0.0163
183	0.087	0.138	-0.0041	0.0065
189	0.027	0.080	-0.0013	0.0038
192	0.057	0.188	-0.0028	0.0091
196	-0.159	0.119	0.0078	0.0058
200	0.013	0.109	-0.0006	0.0053
202	0.071	0.147	-0.0035	0.0073

TABLE VI. The fitted values of ϵ and their 68% confidence level uncertainties together with the 95% confidence level lower limit on Λ and the probability of the Z' signal.

Data set	$\bar{\epsilon}$, TeV^{-2}	$\Lambda_{\min}$, TeV	P, %
All LEP data	0.00037 ± 0.00134	18.6	60.8
$\sqrt{s} \geq 183$ GeV	0.00074 ± 0.00138	17.6	70.4
$e^+e^- \to \mu^+\mu^-$	0.00114 ± 0.00168	15.6	75.2

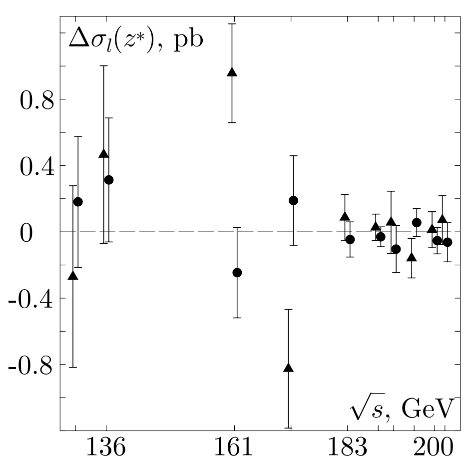

FIG. 1. $\Delta\sigma_l(z^*)$ computed from the LEP data. The circles and triangles represent the measurements for the $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ processes, respectively.